



**Government of Karnataka
Department of Technical Education**

C-25 Diploma Curriculum

Engineering Mathematics For Engineering Programmes

First Semester
(Effect from the AY 2025-26)



Government of Karnataka
DEPARTMENT OF TECHNICAL EDUCATION

Curriculum Structure

I Semester Scheme of Studies- Diploma in _____ Engineering

Sl. No.	Teaching Department	Course Code	Course Name	Hours per week			Total Contact Hours /week	Credits	CIE Marks		Theory SEE Marks		Practice SEE Marks		Total Marks
				L	T	P			Max	Min	Max	Min	Max	Min	
Integrated Courses															
1	SC	25SC11I	Engineering Mathematics-I	4	0	4	8	6	50	20	50	20	-	-	100

L: Lecture: T: Tutorial: P: Practice: SC-Science: T-Theory (Whole Class)::P-Practical(Batch wise)::I-Integrated (Both theory & Practice-Batch wise)

- *For Engineering Mathematics-I, Theory for whole class and Practice batch wise*

Integrated Course Template (T+P)



Government of Karnataka
DEPARTMENT OF TECHNICAL
EDUCATION

Program	Engineering	Semester	I
Course Name	Engineering Mathematics-I	Type of Course	Integrated
Course Code	25SC11I	Contact Hours	8 hours/week (104 hours/semester)
Teaching Scheme	L:T:P - 4:0:4	Credits	6
CIE Marks	50	SEE Marks	50

1. Rationale

The course is designed to give a comprehensive coverage at an introductory level to the subject of Matrices and Determinants, Vectors, Trigonometry, Complex numbers and Limits.

2. Course Outcomes: At the end of the Course, the student will be able to:

CO-01	Solve the system of linear equations using determinants and performs the same in MS Excel.
CO-02	Apply the knowledge extensively in finding product of two vectors and executes the same in GeoGebra graphing calculator tool..
CO-03	Able to solve physical problems using trigonometric ratios and visualize the graphs of trigonometric functions in GeoGebra graphing calculator.
CO-04	Able to solve problems on algebra of complex numbers and interpret the results graphically.
CO-05	Evaluate the limit of a single variable function and extract the limit values for discretized data of a one variable function in MS excel.

3. Course Content:

WEEK	CO	PO (L3- Highly mapped)	Theory (4 Hours per week)	Practice (4 Hours per week)
1	1	1,4,7	MATRICES: -Definition and order of matrices Types of matrices: Row matrix, Column matrix, Zero matrix (Null matrix), Square matrix, Diagonal matrix, Scalar matrix, Unit matrix (Identity matrix)	Practice-1: Introduction to MS Excel
	1	1,4,7	Algebra of matrices: Scalar multiplication and Transpose of a matrix	
	1	1,4,7	Addition and Subtraction of matrices (2x2 only)	Practice-2: Compute addition, subtraction, scalar multiplication of matrices in MS Excel.
	1	1,4,7	Product of two matrices (2x2 only) and Problems	
1	1,4,7	Problems continued		

2	1	1,4,7	DETERMINANTS: Definition, Expansion of determinant of order 2 and Problems	Practice-3: Compute multiplication, transpose of matrices in MS Excel.
	1	1,4,7	Cramer's Rule (Determinant method): Solution of the system of linear equations with two unknowns and Problems	Practice-4: Compute determinant, and inverse of matrices in MS Excel.
	1	1,4,7	Minors, Co-factors, evaluating Adjoint of square matrices explicitly by finding minor and co-factors (2X2 only) and Problems	
3	1	1,4,7	Definition of Singular and non-singular matrices, Inverse of a matrix (2X2 only) and Problems	Practice-5: Solve the system of linear equations by Cramer's rule in MS Excel.
	1	1,4,7	Characteristic equation and characteristic roots of a matrix (2X2 only) and problems	
	2	1,4,7	VECTORS: Definition, notation and types of vectors [Null, Unit, Equal, Coplanar and Collinear vectors]	Practice-6: Installation and introduction to tools in GeoGebra.
	2	1,4,7	Position vector & its magnitude and problems	
4	2	1,4,7	Problems on equilateral, isosceles, right-angled triangle	Practice-7: Finding magnitude of a vector, sum and difference of two vector and visualize it in GeoGebra graph.
	2	1,4,7	Expression and formula for unit vector along the given vector and problems	
	2	1,4,7	Addition and Subtraction of two vectors (Algebraically) and problems	Practice-8: Verifying whether the given three position vectors are vertices of an equilateral triangle in MS excel.
	2	1,4,7	Scalar product (dot product) of two vectors and problems	
5	2	1,4,7	Applications of Scalar product: Cosine of an angle between two vectors and problems	Practice-9: Find the scalar product of two vectors also find the angle between two vectors degrees in GeoGebra. Visualize the dot product of two vectors and hence verify the property of orthogonality.
	2	1,4,7	Condition for two vectors to be orthogonal or perpendicular and problems	
	2	1,4,7	Projection of \vec{a} on \vec{b} and \vec{b} on \vec{a} and problems	Practice-10: Find the work done by the force applied at different angles on the body to move it from point A to B. Hence analyze the amount of work done and give the physical interpretation.
	2	1,4,7	Work done by the vector (force) and problems	
6	3	1,4,7	TRIGONOMETRY: Recapitulation of Trigonometric ratios and identities.	Practice-11: Plot the graphs of trigonometric functions for $\sin x$, $\cos x$ and $\tan x$ in the interval $[-\pi, \pi]$ in GeoGebra.
	3	1,4,7	Define radian of an angle. Conversion of angles (Degree to Radian and Radian to Degree) and Problems	
	3	1,4,7	Allied angles: Definition of allied angle, ASTC Rule	Practice-12: Verify the ASTC rule of quadrants in GeoGebra.

	3	1,4,7	Rules of allied angles ($-\theta, 90^\circ \pm \theta$ & $270^\circ \pm \theta$) and simple Problems.	
7	3	1,4,7	Rules of allied angles ($180^\circ \pm \theta$ & $360^\circ \pm \theta$) and simple Problems.	Practice-13: Construction of clinometer for measurement of sides and angles of a triangle.
	3	1,4,7	Problems continued on Allied angles	
	3	1,4,7	Problems continued on Allied angles	Practice-14: Usage of clinometer (DEMONSTRATION)
	3	1,4,7	Compound Angles: Formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ (without proof) and T-functions of 15° , 75° and 105°	
8	3	1,4,7	Multiple Angles: $\sin 2A$, $\cos 2A$, $\tan 2A$ with proof	Practice 15: Using clinometer find the heights and distances of physical objects in the surroundings.
	3	1,4,7	Multiple Angles: $\sin 3A$ and $\cos 3A$ with proof	
	3	1,4,7	Applications of Trigonometry: Introduction to Heights and Distances	Practice-16: Using clinometer measure the heights and distances of objects in the surrounding.
	3	1,4,7	Problems based only on angle of inclination	
9	4	1,4,7	COMPLEX NUMBERS: - Definition, real and imaginary parts of a complex number $z = a + ib$. Examples	Practice-17: Plot the Cartesian complex numbers z_1, z_2, z_3, z_4 and z_5 . Also plot $z_1 + z_2, z_3 - z_4, 2z_1, z_3/z_4$ and $z_4 \times z_5$ in the graph sheets
	4	1,4,7	Modulus and amplitude of a complex number and Problems	
	4	1,4,7	Conjugate of a complex number and Problems	Practice-18: Plot the polar complex numbers z_1, z_2, z_3, z_4 and z_5 . Also plot $z_1 + z_2$ and $z_3 - z_4$ in the graph sheets
	4	1,4,7	Addition and subtraction of complex numbers and Problems	
10	4	1,4,7	Multiplication of complex numbers and Problems	Practice-19: Generate 50 random data, construct the frequency distribution table and plot Bar chart using MS Excel.
	4	1,4,7	Ratio of two complex numbers and Problems	
	4	1,4,7	Polar form of a complex number and Problems	Practice-20: Generate 50 random data, construct the frequency distribution table and plot Pie chart using MS Excel.
	4	1,4,7	Exponential form of a complex numbers and Problems	
11	4	1,4,7	Conversion of Cartesian form into polar and exponential forms and Problems	Practice-21: Generate 50 random data, construct the frequency distribution table and plot Line graph using MS Excel.
	4	1,4,7	Problems continued	
	5	1,4,7	LIMIT OF FUNCTIONS: Constants and variables, Definition of function. Concept of limits	Practice-22: Generate 50 random data, construct the frequency distribution table and scatter plot using MS Excel.
	5	1,4,7	Evaluation of limits by factorization method and problems	
12	5	1,4,7	Problems continued	Practice-23: Generate 50 random data, construct the frequency
	5	1,4,7	Evaluation of limits by rationalization method and problems	

				distribution table and plot Histogram using MS Excel.
	5	1,4,7	Problems continued	Practice-24: Plot the following functions in GeoGebra and visualize the graphs. i) Odd function ii) Even function iii) Algebraic function iv) Trigonometric functions v) Exponential functions vi) Logarithmic functions
	5	1,4,7	Evaluation of limit of a function of the type $\lim_{x \rightarrow \infty} \left(\frac{f(x)}{g(x)} \right)$ and Problems	
13	5	1,4,7	Problems continued	Practice-25: Using MS Excel, verify that, as x tends to zero the ratio $\frac{\sin x}{x}$ tend to 1, for 20 discrete data in the interval [0.5, 0.1]. (DEMONSTRATION)
	5	1,4,7	Standard Limits (without proof): a) $\lim_{x \rightarrow a} \left(\frac{x^n - a^n}{x - a} \right) = na^{n-1}$, where n is rational b) $\lim_{\theta \rightarrow 0} \left(\frac{\sin \theta}{\theta} \right) = 1$, where θ is in radians c) $\lim_{\theta \rightarrow 0} \left(\frac{\tan \theta}{\theta} \right) = 1$ where θ is in radians d) $\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right) = 1$	
	5	1,4,7	Problems on Standard Limits	
	5	1,4,7	Problems continued	
				Practice-26: Evaluation of limits of standard type in <i>wolfram alpha</i> . (DEMONSTRATION)

4. References:

1. NCERT Mathematics Books for Class XI and XII.
2. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, New Delhi, 40th Edition, 2007.
3. G.B.Thomas, R. L. Finney, Calculus and Analytic Geometry, Addison Wesley, 9th Edition, 1995.
4. V.Sundaram, R.Balasubramanian, K.A.Lakshminarayanan, Engineering Mathematics, 6/e., Vikas Publishing House.
5. Reena Garg & Chandrika Prasad, Advanced Engineering Mathematics, Khanna Publishing House, New Delhi.
6. Online resources (courtesy you tube)
 - i) <https://www.youtube.com/watch?v=wbJcJcKbCmG> – Excel for beginners
 - ii) <https://www.youtube.com/watch?v=RDFb--em5Kg> – construction of clinometer.
 - iii) <https://www.youtube.com/watch?v=tn6UoIz-1yM> – using clinometer.
 - iv) <https://www.geogebra.org/download?lang=en-> to download GeoGebra.
 - v) <https://www.youtube.com/watch?v=RYGBhRN9oHQ&list=PLqZ0eZtMcAlugmcomSSvjPBfewVbX35L7> - Basics of GeoGebra
 - vi) <https://www.youtube.com/@grantsander9529> – More videos on GeoGebra

5. CIE and SEE Assessment Methodologies:

Sl. No	Assessment	Test Week	Duration (minutes)	Max marks	
1.	CIE-1 Theory Test	4	90	50	Average of all CIE=50 Marks
2.	CIE-2 Practice Test	7	180	50	
3	CIE-3 Theory Test	10	90	50	
4.	CIE-4 Practice Test	13	180	50	
5	CIE-5 Portfolio evaluation of all the activities through Rubrics	1-13		50	
Total Continuous Internal Evaluation (CIE)					50 Marks
Semester End Examination (SEE) -Theory			180	100	50 marks (100 marks Scaled down to 50 marks)
Total Marks					100 Marks
Minimum marks to pass in CIE & SEE is 40% individually					

6. CIE Theory Test Model question paper:

CIE 1(at the end of 4th week)

Program	_____ Engineering			Semester	I
CourseName	Engineering Mathematics-I			Marks	50
Course Code	25SC11I			Duration	90 min
Name of the Course Coordinator:					
Section A (Answer any six questions, each question carries 5 marks)					
Q. No.	Questions	CL	CO	PO	
1			1		
2			1		
3			1		
4			1		
5			1		
6			1		
7			1		
8			1		
9			1		
Section B (Answer any four questions, each question carries 5 marks)					
10			2		
11			2		
12			2		
13			2		
14			2		
15			2		

Signature of the Course Coordinator

Signature of the HOD

Signature of the IQAC Chairman

CIE 3(at the end of 10th week)

Program	_____ Engineering	Semester	I
CourseName	Engineering Mathematics-I	Marks	50
Course Code	25SC11I	Duration	90 min
Name of the Course Coordinator:			
Section A (Answer any one question, each question carries 5 marks)			
Q. No.	Questions	CL	CO
1			2
2			2
Section B (Answer any six questions, each question carries 5 marks)			
3			3
4			3
5			3
6			3
7			3
8			3
9			3
10			3
Section C (Answer any three questions, each question carries 5 marks)			
11			4
12			4
13			4
14			4
15			4

7. CIE Practice Test:

Program	Engineering	Semester	I
CourseName	Engineering Mathematics-I	Test	II/IV
Course Code	25SC11I	Duration	3 Hrs
		Marks	50
Name of the Course Coordinator:			
Questions		CO	Marks
a.	OR		50
b.			
Scheme of assessment			
a) Observation			10
b) Conduction			20
c) Result and Output			10
d) Viva			10
II--CIE (ANY ONE QUESTION FROM PRACTICE 1 TO 12(Except 1 and 6)			50
IV--CIE (ANY ONE QUESTION FROM PRACTICE 15 TO 24)			Total Marks

Signature of the Course Coordinator Signature of the HOD Signature of the IQAC Chairman

8. Suggestive Activities:

The List is only shared as an Example and not inclusive of all possible activities of the course. Student and Faculty are encouraged to choose activities that are relevant to the topic and on the availability of such resources at their institution.

Note: Minimum 3 suggested activities should be done.

Sl. No.	Suggestive Activities
01	Application of matrices in coding and decoding.
02	Applications of vectors in dynamics
03	Applications of trigonometry in respective programme domains
04	Plotting circles of different radii ($ z - z_0 = r$), discs ($ z - z_0 \leq r$) and annulus ($R_1 \leq z - z_0 \leq R_2$) in complex plane and record the same in the a document.
05	Evaluation of limits using Wolfram alpha platform.

9. Sample Rubrics for Assessment of Activity (Qualitative Assessment)

Note: Dimension and Descriptor shall be defined by the respective course coordinator as per the activities

Sl. No.	Dimension	Beginner	Intermediate	Good	Advanced	Expert	Students Score
		2	4	6	8	10	
1	Knowledge	Poor knowledge About the subject	Normal knowledge about the subject	Good knowledge about the subject	Very good knowledge about the subject	Excellent knowledge about the subject	8
2	Problems solving ability	Solved minimum number of problems with maximum mistakes	Solved minimum number of Problems	Solved problems with few mistakes	Solved maximum number of problems	Solved all problems in neat manner	10
3	Strategies and Procedure	Hardly uses an effective strategy to solve problems.	Rarely uses an effective strategy to solve problems.	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Typically, uses an effective strategy to solve the problem(s).	Typically, uses an efficient and effective strategy to solve the problems	10
4	Completion	Several of the problem are not completed	Only 30% of the questions are answered correctly	Only 50% of the questions are answered correctly	Only 75% of the questions are answered correctly	All assignment questions are answered correctly	8
5	Neatness and Organization	The work appears sloppy and unorganized. It is hardly to know what information goes together.	The work appears sloppy and unorganized.	The work is presented in an organized fashion but may be hard to read at times.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in a neat, clear, organized fashion that is easy to read.	8
Total marks=8+10+10+8+8=44							44

Equipment/software list with Specification for a batch of 30 students

Sl. No.	Particulars	Specification	Quantity
01	Computers	12 th Generation, Intel Core i3, Graphic card, RAM 16GB, Storage:1TB	30
02	Operating System and software	Windows 10,MS Office, MS excel, GeoGebra	30
03	Internet	High Speed Internet	01
04	Printer	Wireless Multifunctioning printer	03
05	Projector	High resolution, Wi-fi enabled	02
06	UPS	As per standards	5KV

PROBLEMS FOR PRACTICE:

MATRICES AND DETERMINANTS

- If $A = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 6 \\ 1 & 3 \end{bmatrix}$, find the matrix $3A + 2B$.
- If $A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 4 & 1 \end{bmatrix}$, find the matrix $A - 2B$.
- If $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$ then find $A + A^T$ matrix.
- If the matrix $A = \begin{bmatrix} x & 3 \\ 3 & x \end{bmatrix}$ is a singular matrix find the value of x .
- If $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix}$ find AB matrix.
- Solve the system of linear equations by applying Cramer's rule $3x + 2y = 8; 2x + 5y = 9$.
- Solve the equations $x + y = 3; 2x + 3y = 8$ by Cramer's rule.
- Solve the system equations $2x - y = 3; x + 2y = 4$ by determinant method.
- Solve the system equations $2x + 3y = 5; x + 4y = 5$ by applying Cramer's rule.
- If $A = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$ verify that $A(\text{adj}A) = |A|I$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, find AB matrix and also find $(AB)^T$ matrix.
- If $\begin{vmatrix} x & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 0 & x \end{vmatrix}$ find the value of x .
- Find adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- If $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ then find the inverse of the matrix A if it exists
- Find A^{-1} if $A = \begin{bmatrix} 5 & 5 \\ 1 & 2 \end{bmatrix}$
- Find characteristic equation and characteristic roots of the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.
- Find characteristic roots of the matrix $A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$.
- Find characteristic equation and characteristic roots of the matrix $A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$.
- Find characteristic equation and characteristic roots of the matrix $A = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$.

VECTORS

- Find the magnitude of vector $i + 2j + k$.
- If $\vec{a} = i + 2j - k, \vec{b} = 3i - 5j + 2k$ find the magnitude of $3\vec{a} - 2\vec{b}$.
- If $\vec{a} = i + 2j + k$ and $\vec{b} = 2i + 4j - k$ then find $|2\vec{b} - 3\vec{a}|$.
- If $\vec{a} = 2i + j + 2k, \vec{b} = i + 3j + k$ and $\vec{c} = 2i + 2j - k$, find $(\vec{a} + \vec{b}) \cdot \vec{c}$.
- Find the projection of $\vec{a} = i + 2j + k$ on $\vec{b} = 2i - 3j + k$.

6. If the vectors $\lambda i + 5j - 6k$ and $7i + 2j + 4k$ are orthogonal find λ .
7. Find the unit vector of $\vec{a} = 2i + 3j - k$.
8. If $\vec{a} = i + 2j - 3k$, $\vec{b} = 3i - 5j + 2k$, find $\vec{a} \cdot \vec{b}$.
9. If $A = (3, -4, 2)$, $B = (-6, 8, 4)$ then find the position vectors of A and B . Also find \overline{AB} and $|\overline{AB}|$.
10. If $\vec{a} = i + 2j + 3k$ and $\vec{b} = 4i - j - 5k$, find $\vec{a} + \vec{b}$ and $|\vec{a} + \vec{b}|$.
11. If $\vec{a} = 2i - j + k$ and $\vec{b} = 3i + j - k$, find $\vec{a} \cdot \vec{b}$.
12. If $\vec{a} = i + j + 2k$ and $\vec{b} = 2i - j + k$, then show that $\vec{a} + \vec{b}$ is perpendicular to $\vec{a} - \vec{b}$.
13. Find the unit vector of \vec{a} if $\vec{a} = 3i + 4j + k$.
14. If $\overline{OA} = 2i - 3j$ and $\overline{OB} = 8i + 5j$ then find $|\overline{AB}|$.
15. If $\vec{a} = 2i + 5j - 6k$, $\vec{b} = 5i - j + 2k$, find $\vec{a} \cdot \vec{b}$.
16. Find unit vector in the direction of $\vec{a} = 5i - j + 2k$.
17. If the vertices of a triangle have position vectors $4i + 5j + 6k$, $5i + 6j + 4k$ and $6i + 4j + 5k$, then prove that triangle is an equilateral triangle.
18. Show that the position vectors of the points $2i + 3j + 5k$, $3i + 5j + 2k$ and $5i + 2j + 3k$ form an equilateral triangle.
19. If $\vec{a} = 3i - j + \lambda k$, $\vec{b} = 3i + 3j - 4k$ are orthogonal, find the value of λ .
20. Find the cosine of the angle between the vectors $\vec{a} = 2i + 3j - k$ and $\vec{b} = i + 2j + 2k$.
21. Find the cosine of the angle between the vectors $4i - 2j - 3k$ and $2i - 3j + 4k$.
22. Find the cosine of the angle between the vectors $i + j - 3k$ and $2i + j - k$.
23. Find $\cos \theta$ if θ is the angle between the vectors $\vec{a} = 3i - 2j + 5k$ and $\vec{b} = 2i + 3j + k$.
24. Find the cosine of the angle between the two vectors $\vec{a} = 4i - 2j - 3k$ and $\vec{b} = 2i - 3j + 4k$.
25. Find the projection of the vector $\vec{b} = 3i + 5j + k$ on the vector $\vec{a} = 2i + j - 2k$.
26. Find the projection of the vector $\vec{a} = 2i + j - 2k$ on the vector $\vec{b} = 3i + 5j + k$.
27. A particle is acted by constant forces $3i - j + 2k$, $-i + 3j + k$, $i + j - 2k$ and is displaced from the point $(-1, 2, 3)$ to $(2, -1, 5)$. Calculate the total work done by the forces.
28. A force $\vec{F} = 2i + j - 2k$ acting on particle at $(3, 2, 2)$ displaces it to the point $(1, 3, -1)$, find the work done.
29. Find the work done by the force $\vec{F} = 5i + 3j + 7k$ in moving a particle from the point $A(1, 2, -1)$ to $B(3, 1, -4)$.

TRIGONOMETRY

1. Convert 120° into radian and $\frac{3\pi}{2}$ into degree.
2. Find the value of $i. \sin 300^\circ$ $ii. \cot 225^\circ$
3. Prove that $\tan(45^\circ + A) = \frac{1 + \tan A}{1 - \tan A}$
4. Write the formula of $\sin(A - B)$ then find the value of $\sin 15^\circ$
5. Find the value of $\sin 120^\circ \cos 330^\circ - \sin 240^\circ \cos 390^\circ$
6. Simplify $\frac{\cos(360^\circ - A) \tan(360^\circ + A)}{\cot(270^\circ - A) \sin(90^\circ + A)}$
7. Simplify $\frac{\sec(360^\circ - A) \cot(90^\circ - A)}{\tan(360^\circ + A) \operatorname{cosec}(90^\circ + A)}$
8. Prove that $\sin 2A = 2 \sin A \cos A$
9. Prove that $\cos 2A = \cos^2 A - \sin^2 A$
10. Prove that $\cos 2A = 2 \cos^2 A - 1$

11. Prove that $\cos^2 A = 1 - 2\sin^2 A$
12. Prove that $\sin 3A = 3\sin A - 4\sin^3 A$
13. Prove that $\cos 3A = 4\cos^3 A - 3\cos A$.
14. From a point on the ground, the angle of elevation of the top of a building is 30° . If the distance from the point to the base of the building is 50 meters, find the height of the building.
15. A tower is 30 meters high. The angle of elevation from a point on the ground to the top of the tower is 30° . Find the distance of the point from the base of the tower.
16. A tower casts a shadow 20 meters long when the angle of elevation of the sun is 60° . Find the height of the tower.

COMPLEX NUMBERS:

1. Find the value of $1 + i^{10} + i^{20} + i^{30}$
2. Express $\sqrt{3} - i$ in polar form.
3. Express $-1 - i$ in polar form.
4. Find the modulus and amplitude of $\sqrt{3} + i$.
5. Find the modulus and amplitude of $1 - i$.
6. Find the modulus and amplitude of $1 + i$.
7. Express $\frac{(1+3i)}{(1+i)}$ in $a + ib$ form.
8. Express $\frac{(1+3i)}{(1+i)}$ in $a + ib$ form.
9. Express $(1+2i)(3+i)$ in $a + ib$ form.
10. Express $\frac{(2-i)}{(1-i)(3+i)}$ in $a + ib$ form.
11. Find the conjugate of $\frac{(1+i)(1-2i)}{(3+i)}$.
12. Find the modulus and amplitude of $1 + \sqrt{3}i$.
13. Find the real and imaginary parts of $(5+3i)(1-2i)$
14. Find the real and imaginary parts of $(5+3i)(1-2i)$

LIMITS

1. Find $\lim_{x \rightarrow 2} \left(\frac{x^4 - 16}{x - 2} \right)$.
2. Find the value of $\lim_{x \rightarrow -2} \left[\frac{x^3 + 8}{x + 2} \right]$.
3. Find $\lim_{x \rightarrow 0} \left(\frac{3x + \tan 2x}{\sin 3x - 5x^2} \right)$.
4. Evaluate $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{2x^2 - 3x - 4} \right)$.
5. Evaluate $\lim_{\theta \rightarrow 0} \frac{\sin 2\theta}{\sin 3\theta}$.
6. Evaluate $\lim_{\theta \rightarrow 0} \left[\frac{\theta}{\tan 5\theta} \right]$.
7. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{x} \right)$.
8. Evaluate $\lim_{n \rightarrow \infty} \left[\frac{(5-n^2)(n-2)}{(2n-3)(n+3)(5-n)} \right]$.
9. Evaluate $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 3}{x^2 + x + 1}$.
10. Evaluate $\lim_{x \rightarrow -3} \frac{x^2 + 4x + 3}{x^2 + 5x + 6}$.
11. Evaluate $Lt_{\theta \rightarrow 0} \left(\frac{1 - \cos 2\theta}{\theta^2} \right)$.

12. Evaluate $\lim_{x \rightarrow \infty} \frac{5x^2+3x}{7x^2+2x}$.

13. Evaluate $\lim_{x \rightarrow 1} \frac{x^2+5x-6}{x^2-3x+2}$.

14. Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan m\theta}{\tan n\theta}$.

15. Evaluate $\lim_{x \rightarrow 2} \frac{x^2-9x+14}{x^2-4}$.

16. Find $\lim_{\theta \rightarrow 0} \frac{\sin 4\theta}{\tan 5\theta}$.

17. Find $\lim_{x \rightarrow 2} \frac{x^2+5x-14}{x^2+x-6}$.

18. Find $\lim_{x \rightarrow 0} \frac{\sqrt{2+x}-\sqrt{2-x}}{x}$.

19. Evaluate $\lim_{x \rightarrow \infty} \frac{2x^2+3x+5}{6x^2-5x+2}$.

20. Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan 2\theta}{\theta}$.

21. Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}$.

22. Evaluate $\lim_{x \rightarrow \infty} \left(\frac{x^2+x+1}{2x^2-3x-4} \right)$.

23. Evaluate $\lim_{x \rightarrow 1} \left(\frac{2x-2}{x^2-1} \right)$.

24. Evaluate $\lim_{x \rightarrow 2} \left[\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x+2}} \right]$.

25. Find the value of $\lim_{x \rightarrow -2} \left[\frac{x^3+8}{x+2} \right]$.

26. Evaluate $\lim_{x \rightarrow 1} \frac{x^2-2x+3}{x^2+x+1}$.

27. Evaluate $\lim_{x \rightarrow 0} \frac{\sin px}{\tan qx}$.

28. Evaluate $\lim_{x \rightarrow -3} \frac{x^2+4x+3}{x^2+5x+6}$.

29. Evaluate $\lim_{x \rightarrow \infty} \left(\frac{5x^2+3x}{7x^2+2x} \right)$.

30. Evaluate $\lim_{x \rightarrow 1} \frac{x^2+5x-6}{x^2-3x+2}$.

31. Evaluate $\lim_{x \rightarrow 3} \frac{x^3-27}{x-3}$.

32. Evaluate $\lim_{x \rightarrow 2} \frac{x^2-9x+14}{x^2-4}$.